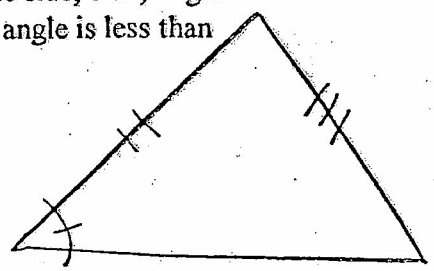
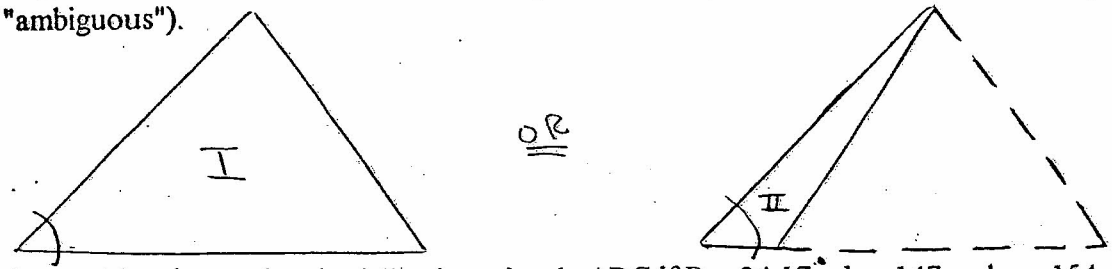
THE CASE OF THE AMBIGUOUS TRIANGLE

You need to check for AMBIGUOUS TRIANGLES whenever you are given the side, side, angle (SSA) of a triangle and the given angle is acute and the side opposite the given angle is less than the ajacent side.



The side opposite the given angle has the potential to "swing" because we don't know the actual length of the bottom side. The result is 2 possible triangle shapes. (Hence, the term "ambiguous").



Our problem is to solve the following triangle ABC if B = 34.17°, b = 147 and c = 154. You will notice when we draw the triangle (below) that we have a case of SSA, so we'll look for 2 triangles (ambiguous).

We will solve these triangles as separate problems.

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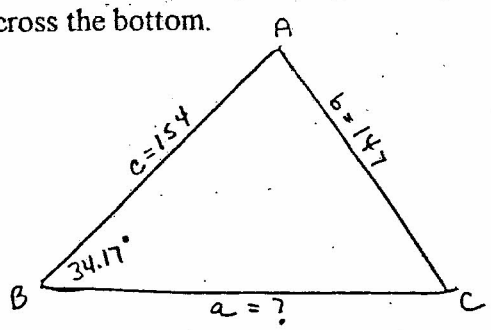
Triangle I:

1) Draw the basic triangle. I always find it easier to "see" if I put the given angle in the bottom left-hand corner and the unknown side across the bottom.



2) Set up a chart.

We need to solve for:

A = ------	a = ------
B = 34.17°	b = 147
C = ------	c = 154

fill in this chart as we go along!!

3) Solve for angle C using the law of sines because we already know side c.

$\frac{147}{\sin 34.17} = \frac{154}{\sin C}$

$\frac{147}{147}(\sin C) = \frac{154(\sin 34.17)}{147}$ -----cross multiply and divide

$\sin C = \frac{154(\sin 34.17)}{147}$

$C = \sin^{-1}\frac{154(\sin 43.17)}{147}$ -----use your calculator

C = 36.04°

4) Find angle A.
We know B = 34.17° and C = 36.04° , so we can find A because a triangle = 180°.

A = 180° - (34.17 + 36.04)°
A = 109.76°

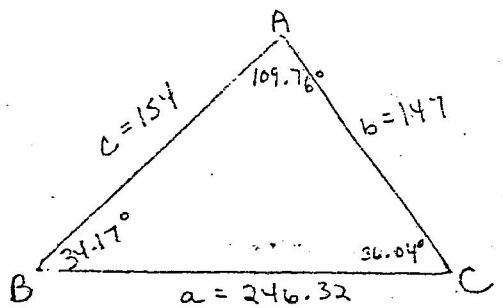
5) Find side a
Since we now know angle A, we can use the law of sines again.

$\frac{147}{\sin 34.17} = \frac{a}{\sin 109.76}$

$a\frac{(\sin 34.17)}{\sin 34.17} = 147\frac{(\sin 109.76)}{\sin 34.17}$ -----cross multiply and divide

a = 246.32

If you filled in the chart on page 1, you now notice that we are done! We have solved the first triangle!!



*note: the angles A and C and side a are approximations due to rounding.

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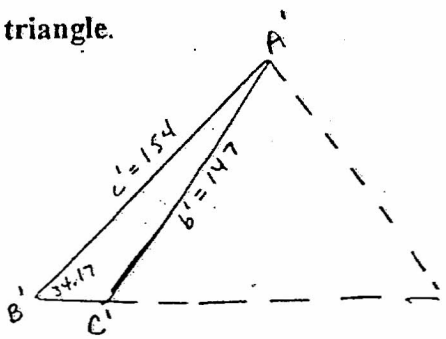
Triangle II:
We now have to solve for the smaller triangle. We'll use A', B', C' to distinguish it from the first triangle. We will follow the same steps as the first triangle.

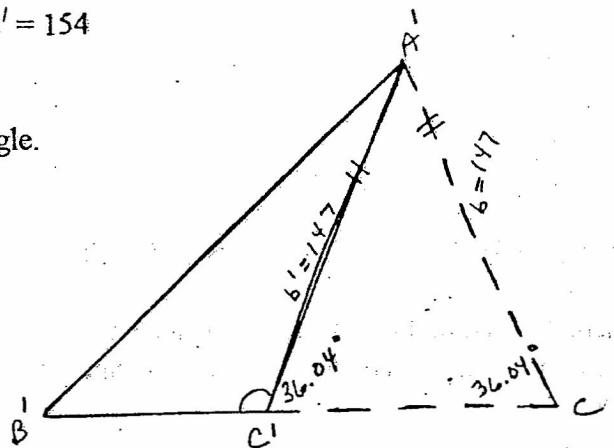
1) Draw the triangle.



2) Set up the chart. (fill it in as we go along)

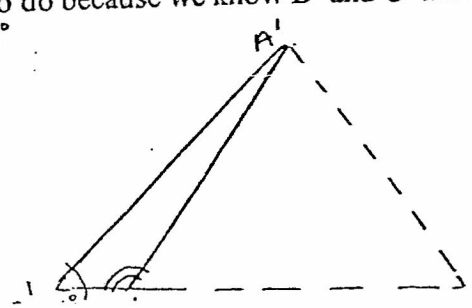
$A' =$ _____	$a' =$ _____
$B' = 34.17°$	$b' = 147$
$C' =$ _____	$c' = 154$

3) Solve for angle C'.
Let's look at the triangle.



- This angle must be $36.04°$ because of the isosceles triangle principle.
  Then, angle $C' + 34.04° = 180°$ because they form a straight line.
  So, $C' = 180° - 36.04°$
  $C' = 143.96°$

4) Find angle A'. ----- easy to do because we know B' and C' and the angles of a triangle $= 180°$
$A' = 180° - (34.17 + 143.96)°$
$A' = 1.87°$



We are almost home-free!!!!!

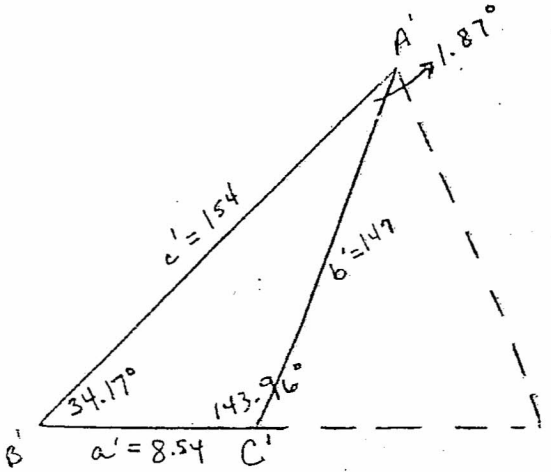
5) Find side a′ using the law of sines.

$$\frac{147}{\sin 34.17} = \frac{a'}{\sin 1.87}$$

$$\frac{a'(\sin 34.17)}{(\sin 34.17)} = \frac{147\,(\sin 1.87)}{(\sin 34.17)}$$ -----cross multiply and divide

$a' = 8.54$

Here is our second triangle:



We are done! We have solved THE CASE OF THE AMBIGUOUS TRIANGLE.

Please take time to do a mental check on both triangles. Make sure that the smallest angle is opposite the shortest side and the largest angle is opposite the longest side. Then, pat yourself on the back for a job well done!